Title: Matrices at the Speed of Light!

Link to Outcomes:

- **Problem Solving** Students will demonstrate their ability to solve application matrix problems using matrices.

- **Communication** Students will cooperatively discuss mathematical concepts and use of technology with other students. They will present their findings in a laboratory notebook or worksheet format.

- **Algebra** Students will demonstrate their ability to write matrix equations and correlate matrices to systems of equations and multiplying polynomials.

- **Technology** Students will develop proficiency in the use of the TI-82 graphics calculator to perform matrix operations.

- **Cooperation** Students will demonstrate the ability to investigate matrix mathematics in groups of two or three.

Brief Overview:

Students will receive detailed instruction on the use of the TI-82 graphics calculator to perform basic matrix operations. To reinforce these concepts, students will solve typical matrix applications using the TI-82 graphics calculator.

Prerequisite Knowledge:

Students should have a good pencil and paper proficiency in the matrix operations of addition, subtraction, multiplication, inverse matrices, scalar operations, and system of equation operations. They should have also completed Algebra I/Course I and Geometry I/Course II.

Grade/Level:

Grades 9 - 12; Algebra II, Selected Topics, Pre-Calculus

Duration:

This lesson is expected to take two to four days depending on discussion, extensions used, and length of the class periods.

Objectives:

- Review basic pencil/paper matrix operations.
- Perform basic matrix operations on the TI-82 graphics calculator.
- Solve matrix application problems using the TI-82 graphics calculator.

Materials/Resources/Printed Materials:

- Teacher generated student worksheets
- Student handout on use of the TI-82 (optional)
- TI-82 graphics calculator

Development/Procedures:

Students will be placed in groups of two at the beginning of the lesson. Using cooperative learning, the students will demonstrate their ability to use the TI-82 graphics calculator by completing the worksheets/problems provided.

Activity 1: Review/Warm Up

1. Teacher briefly reviews basic matrix operations (optional).
2. Students using paper and pencil will complete the worksheet.
3. Students will write a paragraph on each operation explaining how it is performed. (optional)
4. Students will list their likes, dislikes, and why, regarding the use of matrices. (optional)

Activity 2: TI-82 Graphics Calculator Instruction

1. The teacher will demonstrate using the overhead how to set up and perform matrix operations on the TI-82.
2. Each student will perform the matrix operations with the teacher.
3. Students will repeat **Activity 1**, using the TI-82 graphics calculator.
4. Students will be provided an evaluation sheet verifying their understanding of the use of the TI-82 graphics calculator for matrix operations.
5. The teacher will go through the correct answers with the students and collect the papers to verify student comprehension of the process.

Activity 3: Matrix Applications

1. The teacher will review the following matrix applications:
   - Systems of Equations
   - Multiplication of Polynomials
   - SIMPLE Decoding Problems
   - Markov Chains (optional)
2. Students will complete the applications worksheet using the TI-82 graphics calculator.

Evaluation:

The teacher will circulate among the groups to ensure that they are on task. Group evaluations will be based on performance, time on task, quality of discussion, and completion of worksheets. Investigation sheets will be collected and a final evaluation made based on the results of the matrix application problems.

Extension/Follow-up:

A number of extensions are suggested and are attached. Additionally students should be expected to outline how they use the TI-82 graphics calculator to perform each operation.

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USING MATRIX CAPABILITIES OF THE TI-82

Entering a Matrix:

1) Press **<MATRIX>**. Use your right arrow key to select **<EDIT>**. Press **<1>** to enter your matrix, now represented as matrix A.

2) Type in the dimensions of the matrix. Remember to press **<ENTER>** after each entry.

3) Type in the elements of the matrix (numerical rows, left to right). Remember to press **<ENTER>** after each entry.

4) Press **<2nd> <Quit>**

Viewing a Matrix: (previously stored)

Pressing **<MATRIX> <1> <ENTER>** allows you to see matrix A.

<MATRIX> <4> <ENTER> allows you to see matrix D.

Operations on Matrices:

1) Addition: A + B
   Press **<MATRIX> <1> <+> <MATRIX> <2> <ENTER>**.

2) Subtraction: C - D
   Press **<MATRIX> <3> <-> <MATRIX> <4> <ENTER>**

3) Multiplication: A*E
   Press **<MATRIX> <1> <x> <MATRIX> <5> <ENTER>**

4) Scalar Operation: 7*B
   Press **<7> <x> <MATRIX> <2> <ENTER>**

5) Multiplicative Inverse: C^{-1}
   Press **<MATRIX> <3> <X^{-1}> <ENTER>**

6) Squaring a Matrix: D^2
   Press **<MATRIX> <4> <X^2> <ENTER>**

ACTIVITY #1
Warm Up/Review Exercises

A. Given:

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \\ 1 & -3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \qquad E = \begin{bmatrix} -2 & 3 \\ 4 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

Evaluate:

1) A + D

2) E - B

3) 4*C

4) C*B

5) A*D

6) A^{-1}

7) E^2

B. Solve for the variable(s) by using the multiplicative inverse:

8) E*X = B (use E and B from above)

9) $\begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -8 \\ 2 \\ 6 \end{bmatrix}$

10) 2X - Y = 14
3X + Y = -8

ACTIVITY #2:
Evaluation - Use of the TI-82

A. Given:

$$A = \begin{bmatrix} 3 & 0 & 4 \\ -2 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \qquad D = \begin{bmatrix} -2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & -4 \\ 5 & 1 \end{bmatrix}$$

Find each of the following:

1) C + D 4) C*B

2) E - 2*A 5) E^{-1}

3) B*C 6) A^2

B. Use the multiplicative inverse to solve for the variable(s):

7) A*X = B (use A and B from above)

8) $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} * \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$

9) 5X - 2Y = 5
-4X + 3Y = 10

ACTIVITY #3:
Some Applications of Matrices

A. Polynomial Multiplication[1]

Example: $(2X^2 - X + 3)(X^3 - X + 1)$

The degree would be 5. Therefore there are 6 terms in the answer polynomial.

Enter the first polynomial as a 1 x 3 matrix - from the example: $\begin{bmatrix} 2 & -1 & 3 \end{bmatrix}$

Therefore, the second matrix must be a 3 x 6 matrix. It is entered as:

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

The coefficients of the second polynomial are entered in descending order (note the missing term coefficient) and are staggered.

The product of these two matrices is the 1 x 6 matrix, [2 -1 1 3 -4 3], which are the coefficients of the answer polynomial. Therefore, the product is:

$$2X^5 - X^4 + X^3 + 3X^2 - 4X + 3$$

Problems:

Find the products:

1. $(X^2 - 2X + 1)(3X)$
2. $(X - 5)(2X - 7)$
3. $(4X + 7)(X^2 - 2X + 3)$
4. $(5X^2 - 1)(2X^2 - 4X + 1)$
5. $(X^3 - 4X^2 + 3X + 2)^2$

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[1] Advanced High School Mathematics, A Compendium of Topics, The State University of New York, The State Education Department, 1992.

B. **<u>SIMPLE</u> Coding Messages**

Example:

1. Assign numbers to the letters of the alphabet and any other symbols:

   A B C D .... Z /
   1 2 3 4 .... 26 27

2. Select a "coding" matrix. This matrix must be **square**.

   Choose: $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

3. Select a message and using the above codes form a matrix. The number of rows <u>must</u> equal the number of columns of the "coding" matrix.

   a. "Math": $\begin{bmatrix} M & A \\ T & H \end{bmatrix}$ becomes $\begin{bmatrix} 13 & 1 \\ 20 & 8 \end{bmatrix}$

   b. "coding messages":

   $$\begin{bmatrix} C & O & D & I & N & G & / & M \\ E & S & S & A & G & E & S & * \end{bmatrix} \text{ becomes } \begin{bmatrix} 3 & 15 & 4 & 9 & 14 & 7 & 27 & 13 \\ 5 & 19 & 19 & 1 & 7 & 5 & 19 & 0 \end{bmatrix}$$

   Note: * is a "filler" to form a matrix and is replaced with 0.

4. By LEFT operation, multiply the coding matrix by the message matrix.

$$\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} * \begin{bmatrix} 13 & 1 \\ 20 & 8 \end{bmatrix} = \begin{bmatrix} 112 & 28 \\ 46 & 10 \end{bmatrix}$$

5. Send the "coded message," $\begin{bmatrix} 112 & 28 \\ 46 & 10 \end{bmatrix}$

6. To "decode" the message, the receiver must know the "coding" matrix and find its inverse. Using LEFT operation, multiply the inverse with the coded message.

"Coding" matrix: $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

its inverse: $\begin{bmatrix} -.5 & 1.5 \\ 1 & -2 \end{bmatrix}$

Therefore: $\begin{bmatrix} -.5 & 1.5 \\ 1 & -2 \end{bmatrix} * \begin{bmatrix} 112 & 28 \\ 46 & 10 \end{bmatrix} = \begin{bmatrix} 13 & 1 \\ 20 & 8 \end{bmatrix}$

This decodes to $\begin{bmatrix} M & A \\ T & H \end{bmatrix}$, Math

Problems

Decode each of the following messages with the given coding matrix and assigned letter/number relations (ciphers) from the example:

1) Coding Matrix: $\begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$

Coded message:

$$\begin{bmatrix} 5 & 12 & 31 & 9 & -15 & -9 & 19 & -27 \\ 11 & 81 & 64 & 108 & 65 & 39 & 102 & 54 \end{bmatrix}$$

2) Coding Matrix: $\begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & -3 & 1 & 2 \\ -1 & -2 & 2 & 1 \\ 3 & 1 & 1 & -1 \end{bmatrix}$

Coded Message:

$$\begin{bmatrix} 6 & 56 & 14 & 12 & 67 & -2 & 40 & 57 & 32 & 39 & 7 \\ 26 & -26 & 20 & 21 & -12 & 24 & 81 & -31 & 7 & 74 & 22 \\ 40 & -52 & 22 & -2 & -46 & 34 & 2 & -38 & -4 & -15 & 10 \\ 32 & 64 & 26 & 31 & 50 & 52 & 83 & 47 & 103 & 73 & 24 \end{bmatrix}$$

C. **Markov Chains *"Classic Problems"*** (Optional)

1. The Taxi Problem:

   A taxi company has divided the city into 3 regions-Northside, Downtown, and Southside. By keeping track of pickups and deliveries, the company has found that of the fares picked up in Northside, 50% stay in that region, 20% are taken Downtown, and 30% go to Southside. Of the fares picked up Downtown, only 10% go to Northside, 40% stay Downtown, and 50% go to the Southside. Of the fares picked up in Southside, 30% go to each of Northside and Downtown, while 40% stay in Southside. We would like to know what the distribution of taxis will be over time as they pick up and drop off successive fares.[1]

2. Auto Sales Data Representation Problem:

   Three major countries that produce cars for sale in the U.S. are Japan, Germany, and the US itself. When it is time to buy a new car, people will choose a car based in part on the satisfaction they have received from the car they presently own. Suppose that of the car buyers who presently own a U.S. car, 55% will purchase another American made car, 25% will buy a Japanese made car, 10% will buy a German made car, and 10% will buy a car made in none of these countries. Of those who presently own a Japanese car, 60% will buy another Japanese car, whereas 25% will buy American, 10% German, and 5% none of the three. Of those car buyers who own a German car, 40% will again buy a German car, 35% will switch to American cars, 15% will switch to Japanese cars, and 10% will buy from another country. Of those who presently own a car from a country other than the three major producers, 20% will switch to American, 25% will switch to Japanese, 15% will switch to German, and 40% will continue to buy from another country.[1]

   After 30 years of successive purchases of cars, what is the probability of owning an American car, a Japanese car, German car, or a car from another country?

[1] Matrices, National Council of Teachers of Mathematics, 1988.

Answers Activity #1

1. $$\begin{bmatrix} 3 & 3 & -2 \\ 2 & 2 & 3 \\ 3 & -2 & 0 \end{bmatrix}$$

2. $$\begin{bmatrix} -6 & 2 \\ 5 & -1 \end{bmatrix}$$

3. $$\begin{bmatrix} -12 \\ 8 \end{bmatrix}$$

4. CAN NOT be done

5. $$\begin{bmatrix} 7 & 6 & 3 \\ 3 & -2 & 3 \\ 3 & 4 & -1 \end{bmatrix}$$

6. $$\begin{bmatrix} -.5 & .5 & 1 \\ .5 & 0 & -.5 \\ -.5 & 1 & .5 \end{bmatrix}$$

7. $$\begin{bmatrix} 16 & -3 \\ -4 & 13 \end{bmatrix}$$

8. $$X = \begin{bmatrix} -.5 & 5/14 \\ 1 & 4/7 \end{bmatrix}$$

9. $$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 11 \\ -7 \\ 9 \end{bmatrix}$$

10. **X = 6/5 or 1.2**
Y = -58/5 or -11.6

Answers Activity #2

1. $$\begin{bmatrix} 1 & -3 \\ 6 & 4 \end{bmatrix}$$

2. $$\begin{bmatrix} -5 & 2 & -7 \\ 7 & 4 & -1 \\ -1 & 0 & 5 \end{bmatrix}$$

3. $$\begin{bmatrix} 21 & -5 \\ 2 & 5 \\ 10 & 2 \end{bmatrix}$$

4. Error: Dimension Mismatch

5. $$\begin{bmatrix} -.5 & .5 & 1E-14 \\ 1 & -.25 & -.25 \\ -.5 & 0 & .5 \end{bmatrix}$$

6. $$\begin{bmatrix} 13 & 4 & 8 \\ -3 & 2 & -10 \\ 0 & -2 & 6 \end{bmatrix}$$

7. $$X = \begin{bmatrix} 1 & -3 \\ -1.25 & 8 \\ -.25 & 3 \end{bmatrix}$$

8. $$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2 \\ -1.5 \\ -.5 \end{bmatrix}$$

9. X = 5
Y = 10

Answers Activity #3

A. 1. $3X^3 - 6X^2 + 3X + 0$

2. $2X^2 - 17X + 35$

3. $4X^3 - X^2 - 2X + 21$

4. $10X^4 - 20X^3 + 3X^2 + 4X - 1$

5. $X^6 - 8X^5 + 22X^4 - 20X^3 - 7X^2 + 12X + 4$

B. 1. Are you correct

2. Coding is found in the area of discrete math

C. 1. $$\begin{array}{c c} & \begin{array}{ccc} N & D & S \end{array} \\ \begin{array}{c} N \\ D \\ S \end{array} & \begin{bmatrix} .3 & .3 & .4 \\ .3 & .3 & .4 \\ .3 & .3 & .4 \end{bmatrix} \end{array}$$

2. (Entries in the answer matrix has been truncated to the ten-thousandth position)

$$\begin{array}{c c} & \begin{array}{cccc} U.S. & J & G & O \end{array} \\ \begin{array}{c} U.S. \\ J \\ G \\ O \end{array} & \begin{bmatrix} .3703 & .3613 & .1512 & .1170 \\ .3703 & .3613 & .1512 & .1170 \\ .3703 & .3613 & .1512 & .1170 \\ .3703 & .3613 & .1512 & .1170 \end{bmatrix} \end{array}$$